Logical Connectors

Selim Kaan Ozsoy, Middle East Technical University

We have five logical connectors:

- **and**: $\wedge$
- **or**: $\vee$
- **not**: $\neg$ ($\sim$)
- **if, then**: $\rightarrow$
- **if and only if**: $\leftrightarrow$

Truth values of the compound statements constructed by these using the truth tables of these connections.(Wittgenstein)

Given two statements p and q:

1. Conjunction

The conjunction of p and q is the statement $p \wedge q$ and is read as "p and q".
$\rightarrow$ The truth table of $p \wedge q$ is given by:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2. Disjunction

The disjunction of p and q is the statement $p \vee q$ and is read as "p or q".
$\rightarrow$ The truth table of $p \vee q$ is given by:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

3. Conditional

The conditional from p to q is the statement $p \to q$ and is read as:

- "if p, then q"
- "q whenever p"
- "q if p"
- "p only if q"
- "q provided that p"
- "q given that p"
- "p is sufficient for q"
- "q is necessary for p"

$\to$ The truth table is:

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

4. Biconditional

The biconditional from p and q is the statement of $p \leftrightarrow q$ and read as:

- "p if and only if q"
- "p is necessary and sufficient for q"

$\to$ The truth table is given by:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

5. Negation

The negation of p is given by $\neg p$ and is read as "*not* p".

- "it is not the case that p holds"

p	$\neg p$
T	F
F	T

Examples

Write the truth tables of the statements:

1. $p \to (q \to p)$
2. $(p \wedge R) \to (\neg(R \to S))$
3. $((p \to q) \wedge p) \wedge \neg q$

1. $p \to (q \to p)$

p	q	$p \to (q \to p)$
T	T	T
F	F	T
T	F	T
F	T	T

2. $(p \wedge R) \to (\neg(R \to S))$

p	R	S	$(p \wedge R) \to (\neg(R \to S))$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

3. $((p \to q) \wedge p) \wedge \neg q$

p	q	$((p \to q) \wedge p) \wedge \neg q$
T	T	F
T	F	F
F	T	F
F	F	F

This is an example of contradiction.

Definitions

⋆ A statement that is always **true** is called a **tautology**.
⋆ A statement that is always **false** is called a **contradiction**.